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BISTATIC DENIAL USING SPATIAL-TEMPORAL CODING

Research Associates for Defense Conversion, Inc.

Hugh Griffiths

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13. ABSTRACT *(Maximum 200 words)*
This report describes work performed on the design of waveforms emitted by an airborne radar, such that the radar cannot be used by an adversary as a coherent reference for a bistatic radar system. This is accomplished by radiating, in addition to the conventional radar signal, another signal or set of signals, coded so as to be distinguishable from the conventional radar signal, and radiated via different radiation pattern(s) to the conventional radar signal. Thus, in the direction in which the radar operates the masking signal will be suppressed both by the radiation pattern with which it is radiated in that direction, though an adversary will detect a combination of the radar signal and the masking signal, such that he cannot correctly distinguish the radar signal. The investigation included a spatial coding technique based on orthogonal beams from a linear antenna array fed by a Butler Matrix. A simulation code was developed to calculate and plot the auto- and cross-ambiguity functions of arbitrary signals. The results indicate that the level of the masking signal received outside the main beam of the radar signal pattern should be adequate to deny a coherent reference.

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Contents

Summary

This document constitutes a short progress report on work performed under contract no. F30602-98-C-0275, on the subject of the design of waveforms emitted by an airborne radar such that the radar cannot be used by an adversary as a coherent reference for a bistatic radar system. This is done by radiating, in addition to the conventional radar signal, another signal or set of signals, coded so as to be distinguishable from the conventional radar signal, and radiated via different radiation pattern(s) to the conventional radar signal. We refer to this signal (or set of signals) as the *masking signal*. Thus in the direction in which the radar operates the masking signal will be suppressed both by the radiation pattern with which it is radiated in that direction, and by the isolation provided by the coding, so that it should not interfere with the normal operation of the radar. In other directions, though, an adversary will detect a combination of the radar signal and the masking signal, such that he cannot correctly distinguish the radar signal and hence cannot use it as a coherent reference for a bistatic radar. This idea has been termed *spatial denial*, and is an example of a more general concept known as *waveform diversity*.

We propose and investigate a spatial coding technique based on orthogonal beams from a linear antenna array fed by a Butler Matrix, though if the element signals are generated digitally (as would almost certainly be the case) then the Butler Matrix hardware is not necessary. A simulation code has been devised to calculate and plot the auto- and cross-ambiguity functions of arbitrary signals, and this has been validated using various signals of known properties. This has been used to explore some simple examples, using co-channel linear FM chirp signals of opposite slope, and pseudo-random binary sequences. The results indicate that the level of the masking signal echoes in the radar receiver should be well below those due to the real radar signal, whilst the level of masking signal received outside the main beam of the radar signal pattern should be adequate to deny a coherent reference to an adversary, and hence prevent the radar being used as a bistatic illumination source.

Further work is required to explore these ideas in more detail, in particular to explore other kinds of modulation coding, and also to investigate the use of the masking signal to carry useful information.

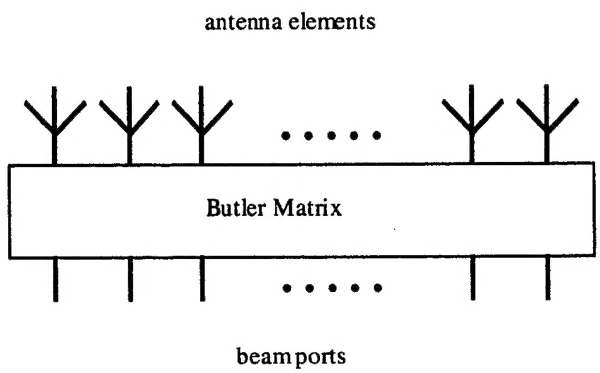
1. Introduction

This document constitutes a short progress report on work performed under contract no. F30602-98-C-0275, on the subject of the design of waveforms emitted by an airborne radar such that the radar cannot be used by an adversary as a coherent reference for a bistatic radar system. This is done by radiating, in addition to the conventional radar signal, another signal or set of signals, coded so as to be distinguishable from the conventional radar signal, and radiated via different radiation pattern(s) to the conventional radar signal. We refer to this signal (or set of signals) as the *masking signal*. Thus in the direction in which the radar operates the masking signal will be suppressed both by the radiation pattern with which it is radiated in that direction, and by the isolation provided by the coding, so that it should not interfere with the normal operation of the radar. In other directions, though, an adversary will detect a combination of the radar signal and the masking signal, such that he cannot correctly distinguish the radar signal and hence cannot use it as a coherent reference for a bistatic radar. This idea has been termed *spatial denial*, and is an example of a more general concept known as *waveform diversity*.

A number of different approaches to this problem have been identified, and are being pursued separately by others within the overall study. One such technique uses a masking signal radiated from an interferometer array. The particular thrust in the work covered in this report has been to consider parameters appropriate to an imaging radar, and to analyze and explore a technique based on a linear array and Butler Matrix processing. This report is organized such that the next section describes this technique in more detail. Section 3 describes a simulation code written to calculate and plot the isolation between waveforms of different codes, as a function of delay and Doppler shift. Section 4 presents and discusses results obtained to date using this code, and Section 5 outlines the direction of future work.

2. Description of 'Butler Matrix' scheme

Consider a sideways-looking radar on an airborne platform, using an N-element linear antenna array. Suppose initially that the array is fed by a Butler Matrix [1]. A Butler Matrix may be considered to be a hardware realization of the Cooley-Tukey Fast Fourier Transform algorithm [2,3].



Linear array and Butler Matrix.

This generates a set of spatially-orthogonal antenna beams, each of the form

$$|E| = \frac{1}{N}\frac{\sin(N\psi/2)}{\sin(\psi/2)} \tag{1}$$

with

$$\psi = \frac{kd}{\lambda}\sin(\theta - \delta) \tag{2}$$

where d is the element spacing, λ is the wavelength, $k = 2\pi/\lambda$, θ is the azimuth angle and δ is the angle of the maximum of the particular beam. For an N-element array

$$\delta_m = \frac{(2m-1)\pi}{N} \tag{3}$$

so the normalized far-field pattern of the mth beam is

$$E_m = \frac{1}{N}\frac{\sin N\{(kd/2)\sin\theta - [(2m-1)/N](\pi/2)\}}{\sin\{(kd/2)\sin\theta - [(2m-1)/N](\pi/2)\}} \tag{4}$$

The orthogonality of this set of beams is maintained over a broad bandwidth, dictated by the hardware of the Butler Matrix, but typically an octave or more. In order for this to be so, the beamwidths and directions of the beams must change with frequency. The beams have a first sidelobe level of –13 dB,

which is rather high for radar purposes. The sidelobe level can be lowered by an amplitude taper across the array in the usual way, but this destroys the orthogonality condition; the extent to which the orthogonality condition may be relaxed to lower the sidelobes is to be explored in future work. The set of beams may be steered electronically by a set of phase shifters, either at the antenna elements or at the beam ports.

Suppose that one of the central beams is used for the radar, both for transmitting and receiving. One or more of the remaining beams is used to radiate the masking signal or signals, at an appropriate relative power level. The problem, then, is one of finding suitable designs for the radar signal and masking signal waveforms, such that the combination of (i) the effects of the radiation patterns described above, and (ii) the waveform codings give sufficient suppression of the masking signal in the radar receiver so as not to disrupt the radar operation, whilst at the same time giving a just sufficient level of masking signal in other directions to disrupt proper reception by an adversary of the radar signal. Based on knowledge of the effect of ECM on imaging radar we can say that a masking signal to radar signal ratio of about 3 dB will cause considerable disruption of the reception of the radar signal. Once the ratio reaches about 13 dB the radar signal will be almost completely obliterated.

In order to evaluate the effects of the waveform codings, we need to take into account the Doppler shift of the radar and masking signal waveforms caused by the radar platform motion, and potentially also by target motion as well. This has led to the formulation of the cross-ambiguity function of the radar and masking signals, and the development of a simulation code to calculate and plot cross-ambiguity functions, which are described in the next section.

A further idea to be investigated is whether it may be possible to use the masking signal to carry useful information, such as telemetry of imagery or target detections to a ground station. Some initial work on this is reported in the penultimate section.

As a final comment, we realise that if the radar signal and masking signal(s) are designed suitably, and if they were to be generated at the beam ports of the Butler Matrix by direct digital synthesis, which could include the effect of phase shifts to steer the beams electronically, then since the signals radiated from each element are simply weighted combinations of the beam port signals, the element signals may be calculated and generated directly, without any need for the Butler Matrix hardware.

3. Simulation code

3.1 Theoretical basis

The performance of a radar waveform can be quantified, in terms of resolution, sidelobe structure, and ambiguities, in range and Doppler domains, by means of the *ambiguity function*, which was originally conceived by Woodward [4]. This plots the point target response of the radar as a function of delay (equivalent to range) and Doppler frequency (equivalent to velocity), by calculating the response of a matched filter for the waveform $u(t)$, to an echo of delay τ and Doppler shift ν:

$$\chi(\tau,\nu) = \int_{-\infty}^{\infty} u(t)u^*(t-\tau)\exp(j2\pi\nu t)\,dt \tag{5}$$

The ambiguity function is defined as the square magnitude of this :

$$|\chi(\tau,\nu)|^2 \tag{6}$$

For our purposes, we are interested in the response of the radar receiver to an echo (from a target or from clutter) from the masking signal. This leads to the definition of the *cross-ambiguity function*, introduced by Rihaczek [5] :

$$|\chi_{12}(\tau,\nu)|^2 = \left|\int_{-\infty}^{\infty} u_1(t)u_2^*(t-\tau)\exp(j2\pi\nu t)\,dt\right|^2 \tag{7}$$

which is the response of a filter designed to be matched to waveform $u_1(t)$ (i.e. the radar signal), to a signal $u_2(t)$ (i.e. the masking signal).

The next subsection (3.2) describes the structure and use of a MATLAB simulation code which has been written to calculate and plot the cross-ambiguity functions of arbitrary waveforms. Subsection 3.3 presents the results of some tests carried out with signals whose auto- and cross-ambiguity properties are known, to validate the simulation code.

3.2 Structure

The software was written to calculate and plot the auto-ambiguity and cross-ambiguity functions of different waveforms. In this way we have been able to check the behaviour of specific waveforms (linear frequency modulation, pseudo-random binary sequences, phase codes, etc.). The cross

ambiguity function also allows us to evaluate the effect of the radiation pattern of the antenna by setting one signal to be the radar signal and the other signal to be the masking signal.

The software is comprised of 7 m-files and is run in MATLAB environment. What follows are the most important parts of these m-files, which show how the theory that was explained in the previous section is implemented in MATLAB code.

3.2 1 Ambiguity function

The following part comes from the Ambiguity.m function (lines 150 – 164) and calculates the ambiguity function of two signals (auto-ambiguity function if both signals are the same and cross-ambiguity function for two different signals). The command "conv" stands for the convolution of the two signals. At the end the ambiguity function is also calculated in dB.

```
A = zeros(length(delay),length(doppler));
for k = 1:length(doppler),
A(:,k) = conv(signal1.*exp((j*2*pi*doppler(k)).*t),signal2).';
end
A = abs(A);
if isempty(A) == 0,
A = A/max(max(A));
end
AdB = 20*log(A)/log(10);
```

3.2.2 Generation of different signals

The software package can have as its inputs any kind of signals. It can actually generate by itself three common types of signal modulation : linear frequency modulation, pseudo-random binary sequences, and P-codes. A few smaller sub-programs were written for the generation of these signals. As an example we describe below the one which generates linear FM chirp signals :

A linear FM waveform may be expressed as :

$$u(t) = a(t)\cos\left[2\pi\left(f_0 t + k/2t^2\right) + \phi\right] \tag{8}$$

Here, $a(t)$ is a pure amplitude modulation function and k is a constant which decides the chirp rate and hence the pulse compression ratio. f_0 denotes the carrier frequency. For a chirp signal of time duration T and bandwidth B, k is given as B/T. The instantaneous frequency is found by differentiating the phase with respect to time, which results in

$$f(t) = (1/2\pi)[d(2\pi kt)/dt] = kt \tag{9}$$

In many cases it is convenient to adopt complex notation to express a signal. If the amplitude envelope *a*(*t*) is simply taken as an unweighted rectangular signal with time duration *T*, and discarding the phase term then *u*(*t*) is expressed in a complex form as

$$u(t) = \mathrm{rect}(t/T)\exp\left[j2\pi\left(f_0 t + Bt^2/2T\right)\right] \tag{10}$$

What follows is a part from the SignalCal.m function (lines 19 – 27) which is used by the software to implement the above:

```
case 'LFM',

if (Signal.TimeRes <= 0) |(Signal.TimeRes > (Signal.TimeMax-Signal.TimeMin)),

return;

end

B = Signal.FreqStop - Signal.FreqStart;

TimeRange = 0:Signal.TimeRes:(Signal.TimeMax-Signal.TimeMin);

PulseDuration = Signal.TimeMax - Signal.TimeMin;

S(kMin:kMax) = exp(j*pi*B*((TimeRange).^2)/PulseDuration+ j*2*pi*Signal.FreqStart*TimeRange);
```

3.2.3 Weighting techniques

The software allows the use of weighting functions to lower the sidelobes. Harris devised a three and then a four term Blackman-Harris windows, as follows:

$$W_{BH}(f) = a_0 + a_1\cos(2\pi f/B) + a_2\cos(2\pi 2f/B) \quad \text{and} \tag{11}$$

$$W_{BH}(f) = a_0 + a_1\cos(2\pi f/B) + a_2\cos(2\pi 2f/B) + a_3\cos(2\pi 3f/B) \tag{12}$$

With an appropriate choice of the parameters $\{a_1,\ldots,a_4\}$, the Blackman-Harris window achieves up to –92dB of sidelobe suppression. In our software the parameters used are:

For the three-term window:
$a_1 = 0.42323$
$a_2 = 0.49755$
$a_3 = 0.07922$

For the four-term window :
$a_1 = 0.35875$
$a_2 = 0.48829$
$a_3 = 0.14128$
$a_4 = 0.01168$

What follows is a part of the Ambdialog.m function (line 391-401) which is used by the software to implement the above:

```
'case 1,',...
      'W = 1;',...
'case 2,',...
      'p = -0.5:1/(length(t)-1):0.5;',...
      'W = 0.42323+0.49755*cos(2*pi*p)+0.07922*cos(4*pi*p);',...
'case 3,',...
      'p = -0.5:1/(length(t)-1):0.5;',...
      'W = 0.35875+0.48829*cos(2*pi*p)+0.14128*cos(4*pi*p)+0.01168*cos(6*pi*p);',...
'end,',...
'F = F .* W;',..
```

Case 1 stands for the non-weighted case ($W = 1$).

Case 2 stands for the three-term Blackman-Harris window.

Case 3 stands for the four-term Blackman-Harris window.

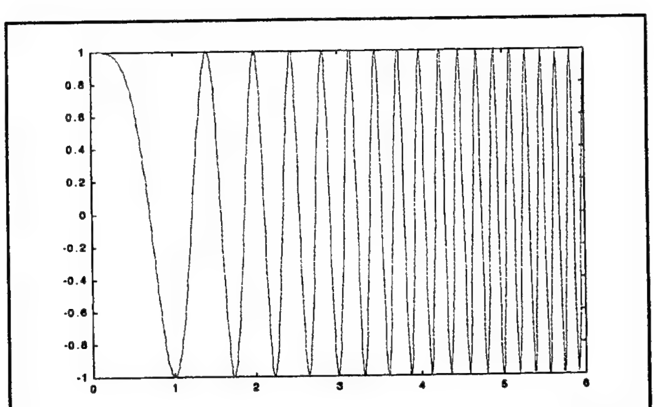
As an overall result we can say that the software package takes two signals pre-generated by the actual software (or otherwise generated), calculates their ambiguity function (auto or cross depending on the signals), weights the result with a Blackman-Harris window, and finally gives a 3-Dimensional graphical representation, as well as a numerical calculation of their ambiguity function.

3.3 Validation

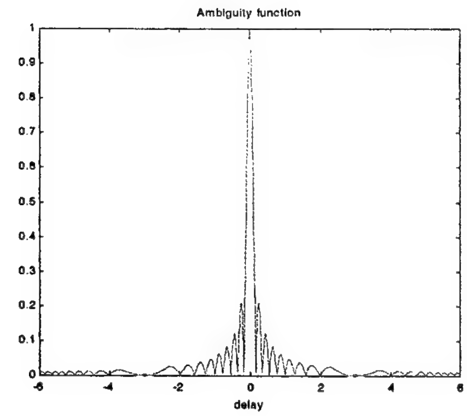
To be able to check whether the above software was working appropriately we needed to reconfirm the obtained results with already known results. We describe below some results which confirm the correct operation of the code.

3.3.1 Checking the auto-ambiguity function of known signals

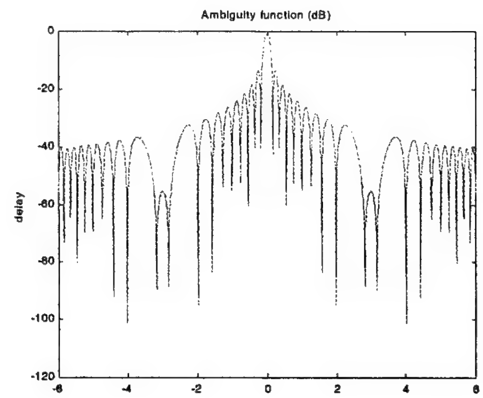
We checked the auto ambiguity function of some linear FM signals. The results were exactly as expected.



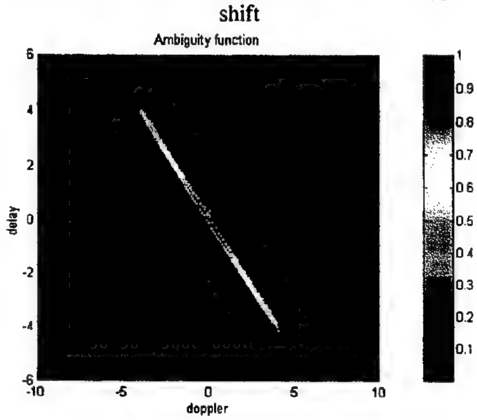
Chirp signal in the time domain.



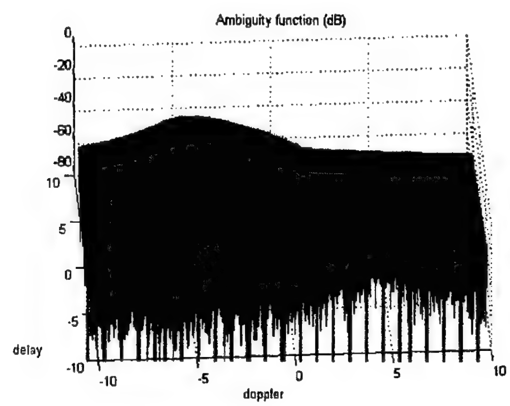
Ambiguity function (linear phase) for zero Doppler shift



Ambiguity function in dB for zero Doppler shift

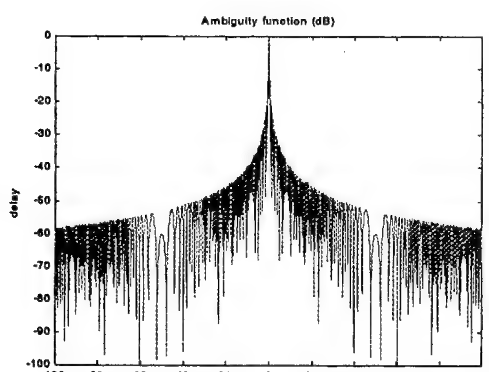


Ambiguity function (linear scale) looking from above

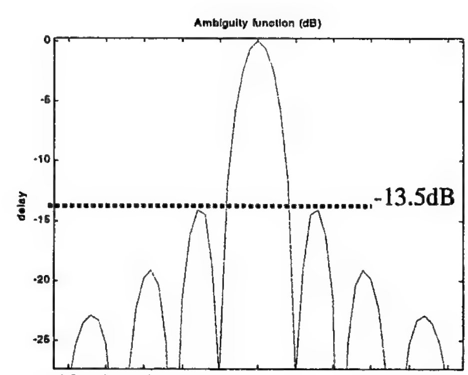


Ambguity function in dB

Although the linear FM waveform achieves satisfactory results and is widely used, it is not without its disadvantages. The compressed pulse has a relatively high sidelobe level, around –13.5dB. An expanded view of the close-in sidelobe structure shows the following, as expected. The same result was obtained for several different types of chirp signals.



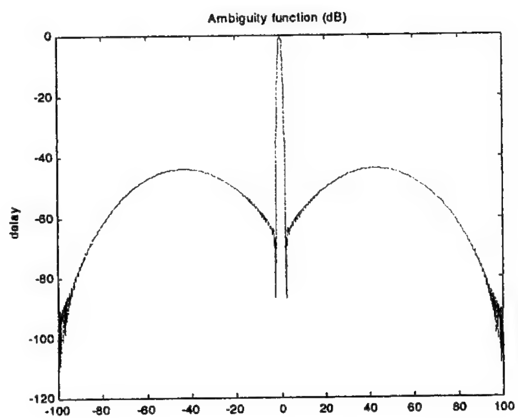
Ambiguity function of a chirp signal at zero Doppler



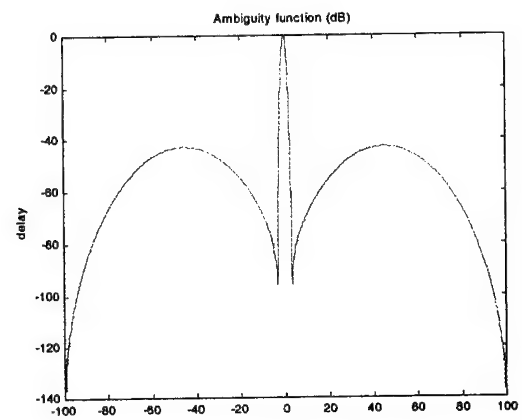
First sidelobes are at –13.5dB

3.3.2 Checking the weighting schemes as a means to reduce the sidelobe level

For this test we have used Linear FM signals. We compared our results with the results found by Cook and Paolillo [6] and by Griffiths and Vinagre [7]. The graphs obtained using our algorithm are shown in the figure below. We take a case of zero Doppler shift because the weighting effect is more obvious on a 2 Dimensional graph than on a 3 Dimensional one.



3-term Blackman-Harris window



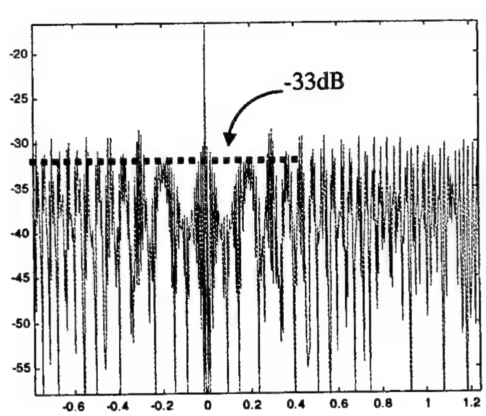
4-term Blackman-Harris window

These results are identical to those reported in references [6] and [7]; this was the case for all of our tests with different signals, using both three and four term Blackman-Harris windows.

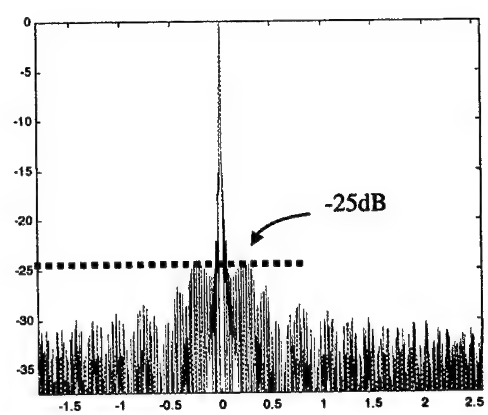
3.3.3 Checking the cross-ambiguity function of chirp signals of opposite slope

The cross-ambiguity functions of co-channel linear FM chirp signals of opposite slope have been evaluated by Giuli *et al.* [8]. Since such signals are an obvious simple choice for the modulation of radar an masking signal waveforms, it is sensible to check that our simulation code gives the same results as reference [8].

The investigation was done for different values of time-bandwidth product BT. The graphs obtained using our software are shown below; the first is for $BT = 1000$ and the second for $BT = 200$.



Cross-ambiguity function for chirp signals of opposite slope (BT = 1000)

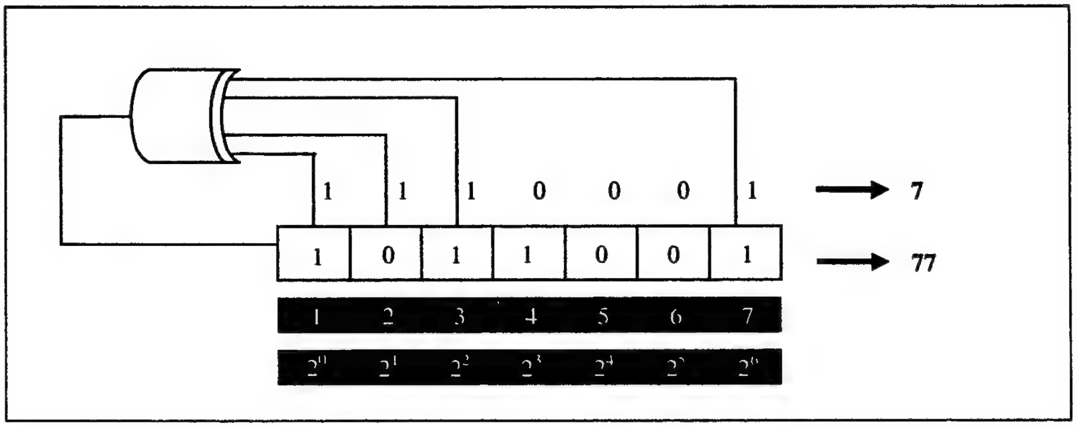


Cross-ambiguity function for chirp signals of opposite slope (BT = 200)

For the first case reference [8] gives the level of the first sidelobes as –33dB; in the second case the level should be –25dB. Our results were identical, confirming the correct operation of our code.

3.3.4 Checking the auto-ambiguity function of more complex signals (PRBSs)

In the first part of this section we checked the auto-ambiguity function of chirp signals. Previous investigation has shown that for our bistatic radar situation some other signals apart from linear FM could be used. One such group of signals are maximal-length pseudo random binary sequences (PRBSs). To be able to check the auto-ambiguity function and therefore the performance of these sequences first of all we need to generate them. This was done as follows, referring to the figure below.



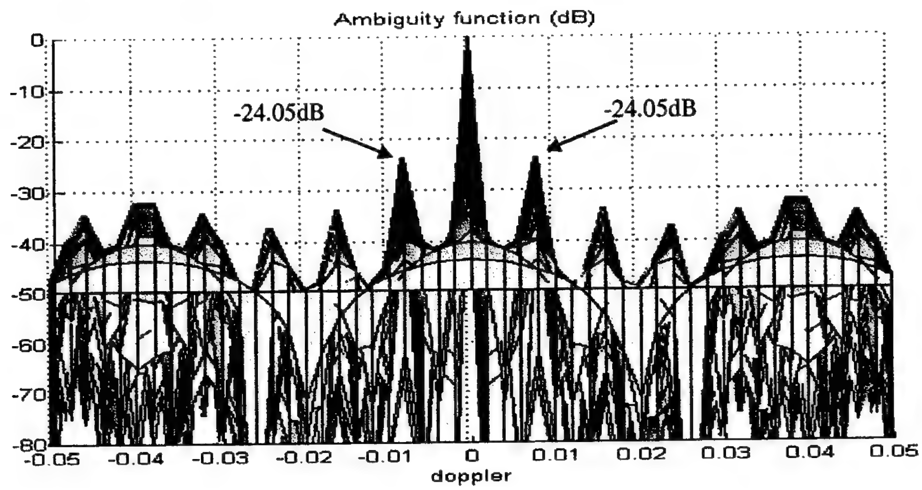
There are taps at positions 1, 2, 3 and 7 of the shift register and the initial loading is 1001101. Postions 1, 2, 3 and 7 denote 2^0, 2^1, 2^2, 2^6. The shorthand notation adopted is (7,77). The first element stands there because $2^0+2^1+2^2=7$. The number 77 is the decimal representation of the binary number 1001101. The tap at the last stage of the register is ignored for these purposes since it must always be one of the feedback taps. To implement the sequence represented for example by (7,127) we should do the following:

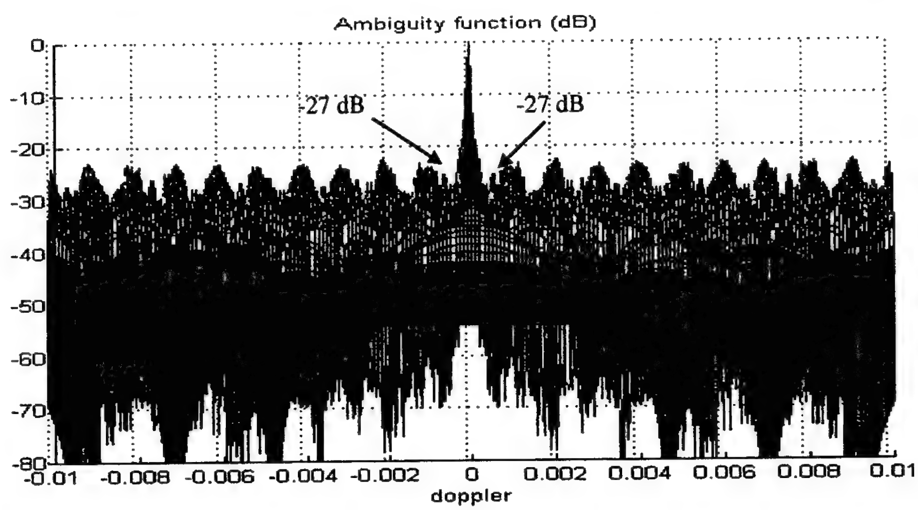
First find what is the number 7 in binary: 111(command *dec2bin(7)*). Then flip that number from left to right: 111(command *fliplr*). Pad that number with zeros until its length equals the length of our initial sequence minus one. The number 124 in binary is represented by a seven bit number thus we should pad 111 with another 2 zeros (11100). Finally add number one at the 7th digit since this tap is always one of the feedback taps. What we get would look like 1110001 and means that there are taps at positions 1, 2, 3, 7. Using that information in combination with the initial loading of the shift register the final maximal length sequence is produced.

Griffiths *et al.* [9] reported the auto- and cross-ambiguity performance of PRBS codes of length 128, 256, and 512. Their results showed that for different sequences the first sidelobes had different values.

For the sequence (115,115) of length 255 the peak sidelobe level was -24.05dB. For the sequence (234,413) of length 511 the peak sidelobe level was at -27dB.

Using our algorithm we tried to reconfirm the above and the results we got are shown below. It is once more obvious that the results are as expected i.e., identical to the ones in reference [9].



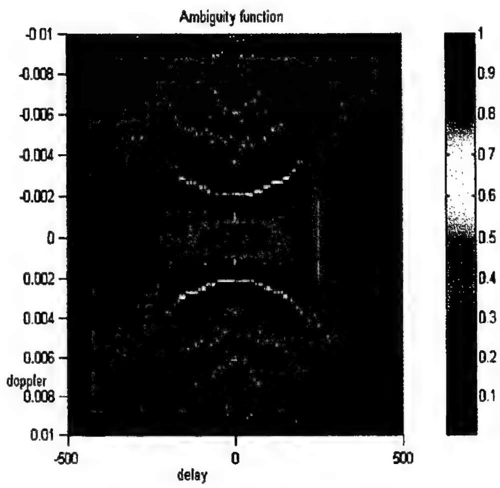


Notice that for both examples the level of the sidelobes is almost the same throughout the whole Doppler range.

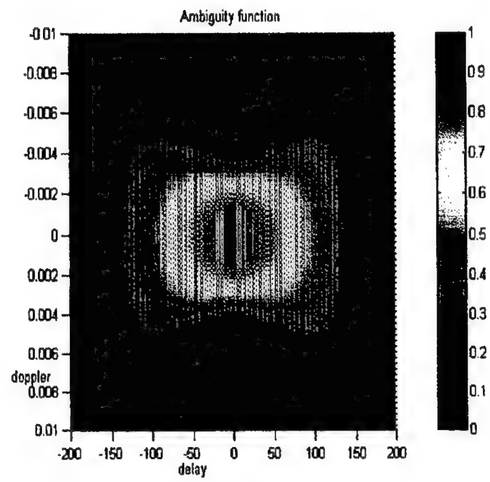
3.3.5 Checking the relationship between Doppler tolerance and code length

There was a suggestion that for PRBS codes the fall off of the main lobe with Doppler shift depends on the code length, more specifically that the main sidelobe falls at a Doppler shift which is approximately equal to 1/(code length). Using our algorithm we checked the above and the results are shown below.

We used two different sequences: the first was the sequence (115, 209) of length 255 with minimal extremity of the time signal range of 0μs and maximal extremity of the time signal range of 500μs. We would expect the mainlobe fall off at around 0.002MHz or 1/500μs. The second was the sequence (115, 115) of length 255 with minimal extremity of the time signal range of 0μs and maximal extremity of the time signal range of 200μs. For this one we would expect the mainlobe fall off at around 0.005MHz or 1/200μs.



Sequence (115,209) of length 255



Sequence (115, 115) of length 255

As seen from the graphs the Doppler tolerance does indeed depend on the length of the code, such that the mainlobe falloff is equal to 1/(code length).

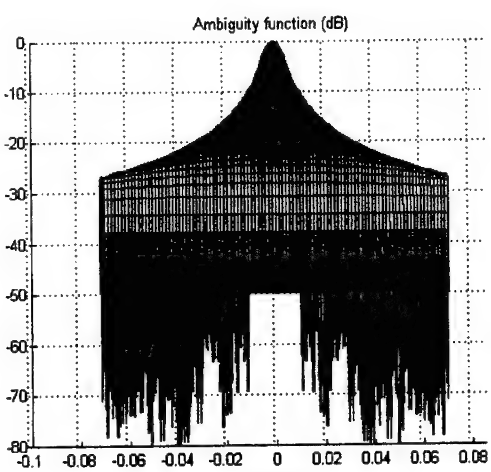
4. Results and discussion

4.1 Peak signal to sidelobe ratio

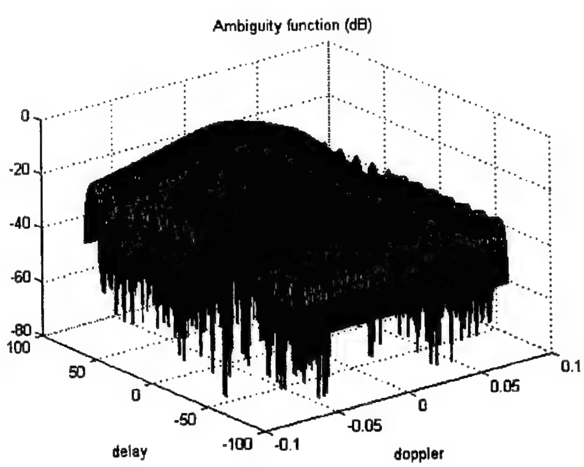
Peak sidelobe level is one of the most important factors in waveform design. To lower the sidelobes, amplitude weighting functions are conventionally used. In such a case the receiver pulse compression filter is no longer strictly a matched filter, which involves a mismatch loss penalty which will lower the peak power at the compression output. The lower the peak sidelobe level, the wider the dynamic range of detectable targets becomes. In fact most of the performance improvement schemes for pulse compression focus on reducing this undesired peak sidelobe level, at the same time trying to minimise side effects such as mainlobe broadening. The following section presents an analysis of pseudo random binary sequence and P-code signals in terms of their peak to sidelobe ratio, before and after applying improvement schemes.

4.1.1 Pseudo-Random Binary Sequence signals (maximal length)

Although its sidelobes are not very low the code length is not limited, which makes these codes attractive for radar applications. The peak sidelobe level is not precisely related to the length of the pulse codes. Some of the sequences of this kind have very good characteristics and others do not. To find a maximal length sequence code with relatively good performance is not straightforward. From the results of the sequences checked we conclude that the autocorrelation function has a relatively flat sidelobe pattern with its peak level around –30dB. The strong sensitivity to Doppler shift means that the application area is very limited. Finally we may say that it shows favourable performance for waveform diversity application. What follows are some results obtained for the maximal length sequences (115,115) of length 255 and maximal extremity of time signal range of 100μs.

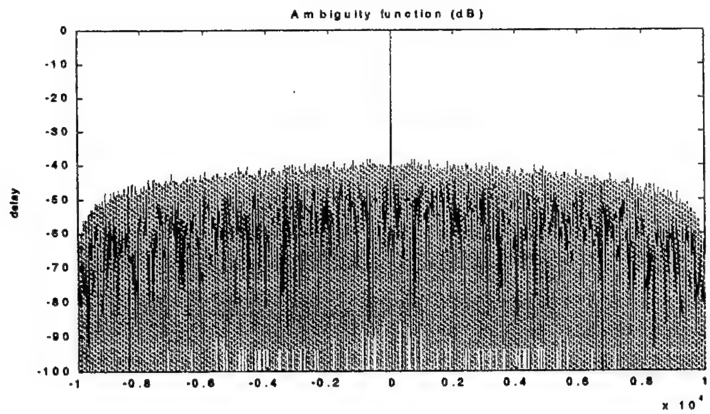


Ambiguity function of the sequence (115,115) of length 255.



Ambiguity function of the sequence (115, 115) of length 255 but this time seen from a different angle.

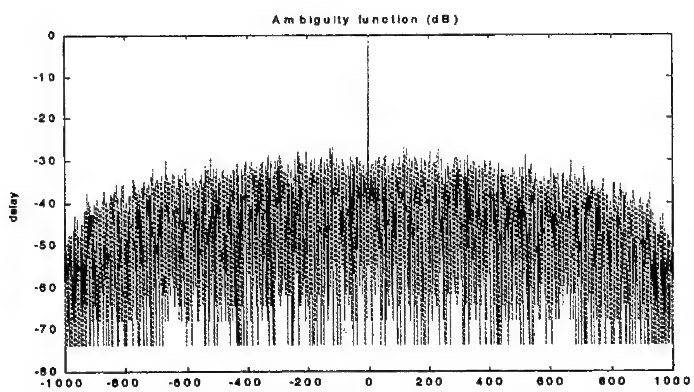
What follows is very interesting. Although we have been dealing with maximal length sequences of length 127, 255 or 511 we tested a sequence which uses a 42 cell shift register, in other words of length of 100000.

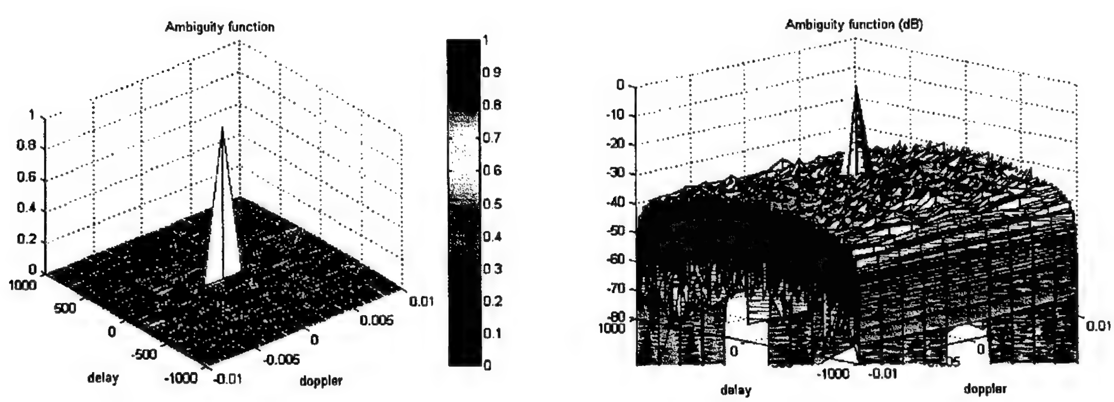


Ambiguity function in dB

From the graph we can see that the sidelobe level is very low at –45dB, which is why we have investigated this specific type of long sequences further.

To be able to run the algorithm faster we used a shorter sequence of length 10000.

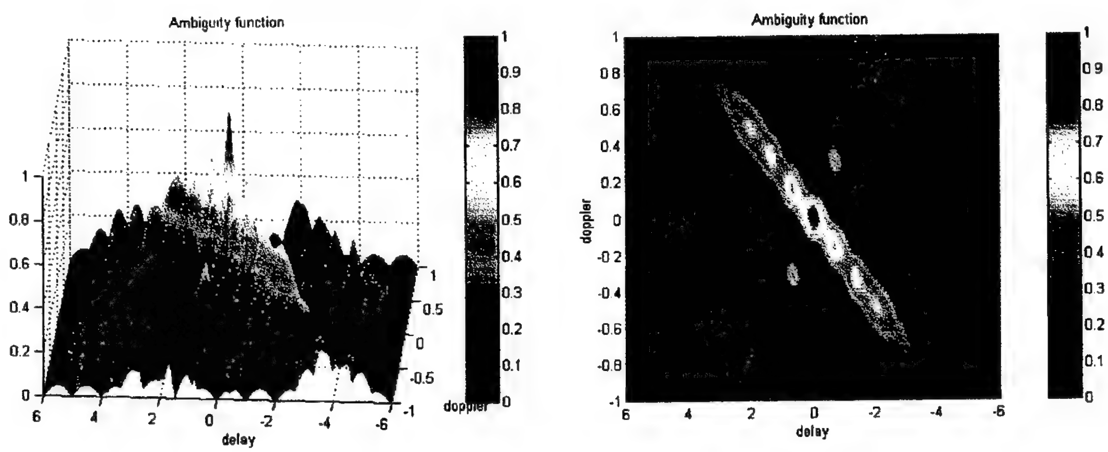




Auto-ambiguity functions at zero Doppler shift (2D), linear scale and in dBs (3D)

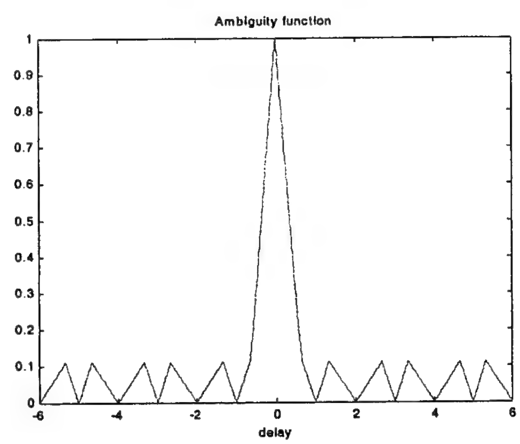
- *P-code Signals*

The general sidelobe characteristics of the Kretschmer and Lewis P-code sequences [10] do not show significant advantages over those of maximal length sequences. But when a slight Doppler shift is introduced in the waveform some of them show strong tolerance. Because phase coded signals are derived from linear FM it might be supposed that they share similar characteristics. After pulse compression the peak sidelobe level of the P-code signals appears to be much lower compared to the linear FM case. The –13.2dB peak sidelobe level does not occur with the P-codes. Instead the sidelobe curves trace the lower pitches of the linear FM sidelobe pattern. The disadvantage of the P-codes ambiguity function is that the mainlobe width is increased. Also, the sidelobes are not fully optimised since they do not form a uniform pattern.

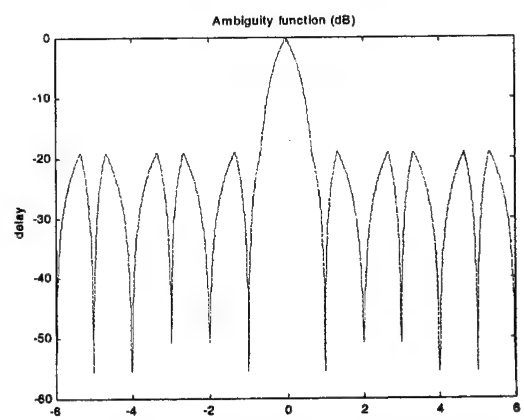


Ambiguity functions seen from different angles

From the above graphs we can see that the behavior of the P-code signals is similar to that of linear FM chirp signals. The ambiguity function looks very similar to that of the chirp signals.



Ambiguity function (linear scale) for zero Doppler shift. Notice that the width of the main lobe is increased.



Ambiguity function (dB) for zero Doppler shift.

4.2 Effect of antenna radiation pattern

All the examples seen so far have only considered the ambiguity behavior of the signals themselves, and have not taken into account the effect of the antenna beam patterns. The following section shows how this can be taken into account and presents some initial results.

We investigated first the case where both signals have equal amplitudes, and then gradually reduced the amplitude of the masking signal until it reached the other extreme, i.e. where its amplitude is very low compared with that of the radar signal.

The results obtained are as follows:
(graphical proofs of these examples follow)

Amplitude of masking signal is the same as that of the radar signal.

The signal can be detected only around the very close region to the origin (in the graph this is zero). The values after that are at –25 to –30dB which is the case because the masking signal is overwriting the radar signal.

Amplitude of the masking signal is at the level of the first sidelobe of the radar signal.

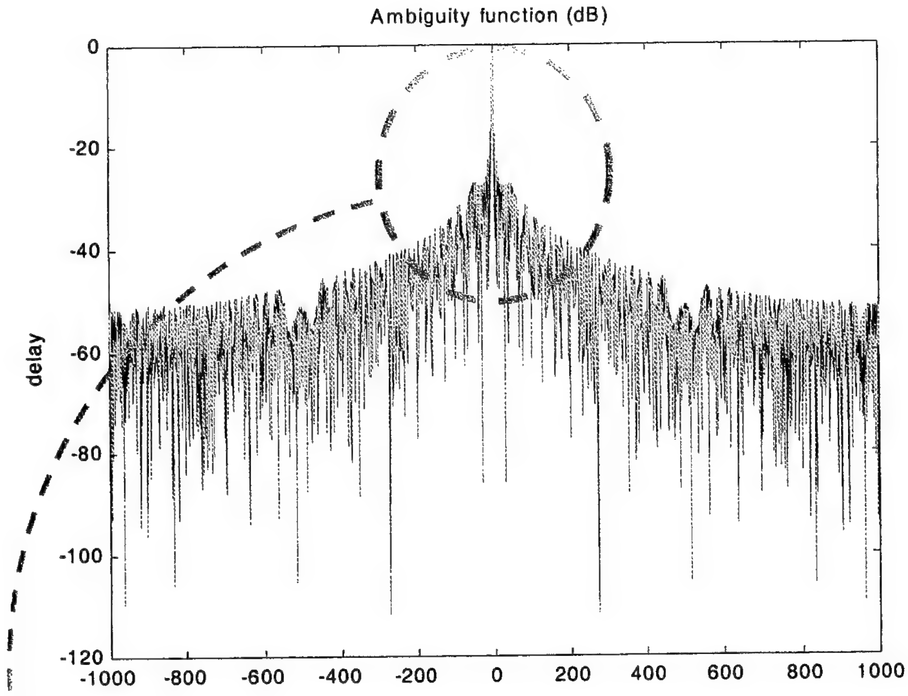
The radar signal can be detected at a longer distance from the origin and this is seen from the fact that this time the value is at –20dB.

Amplitude of the masking signal is negligible in comparison with that of the radar signal.

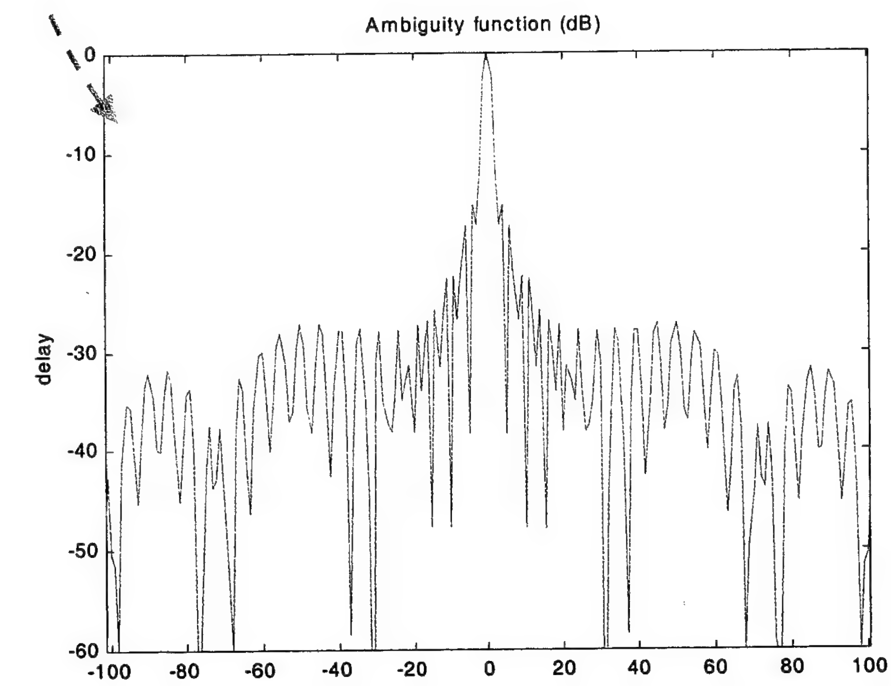
The radar signal can be detected in all directions and this is seen from the graphs as the value is at around –6dB, almost constant around the origin; further out it gradually goes down to –17dB, because the amplitude of the sidelobes of the radar signal at that stage are by themselves at very low levels.

Amplitude of the masking signal lies in between the radar's mainlobe and first sidelobe.

The signal can be detected at the close area around the origin. Then because the masking signal has higher amplitude mainlobes than the radar's first sidelobe the value falls dramatically to –35 to –40dB but then as the amplitude of the radar's and masking signal's sidelobes are approximately similar, the value again goes up to around –30dB, which is as expected.

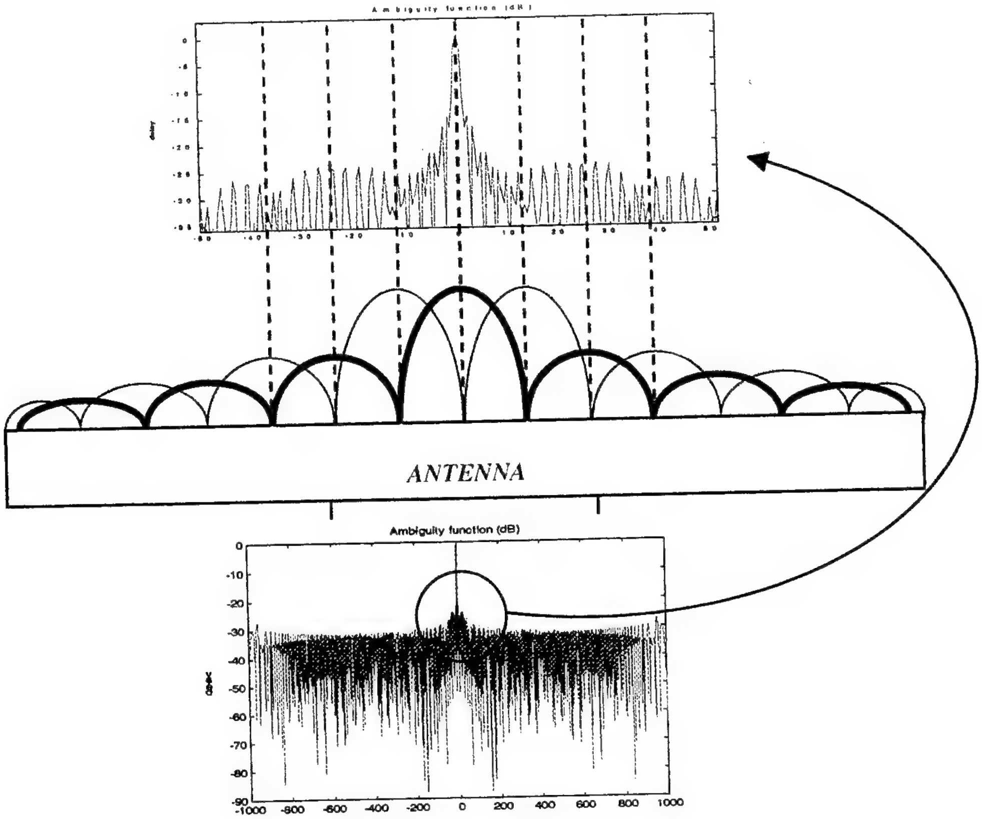


A linear FM signal and an expanded version of it.

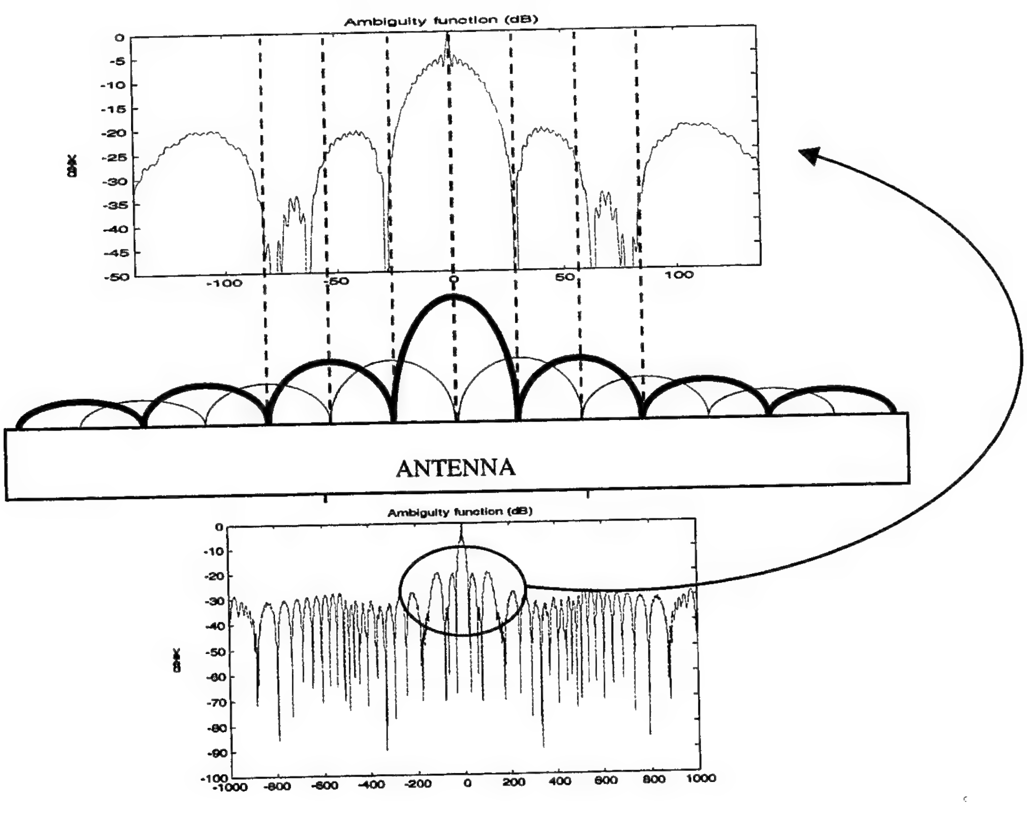


The first sidelobes occur at approximately –30dB. Gradually they go down to –53dB.

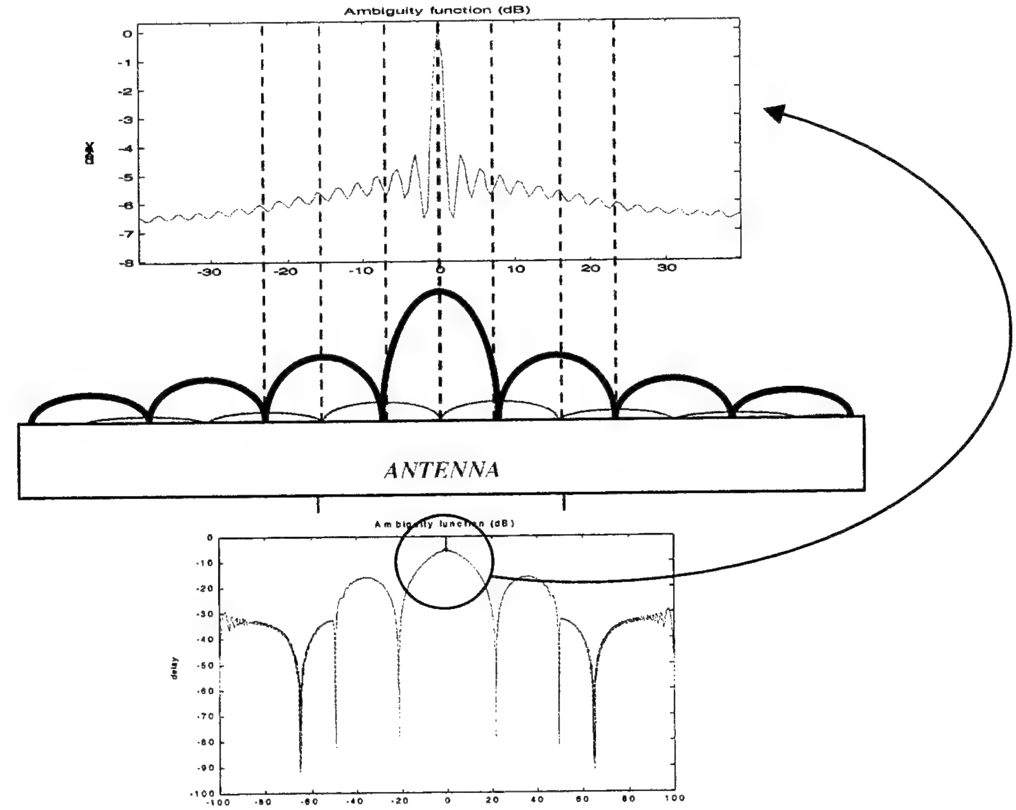
CASE 1 : *Amplitude of masking signal is the same as that of the radar signal.*



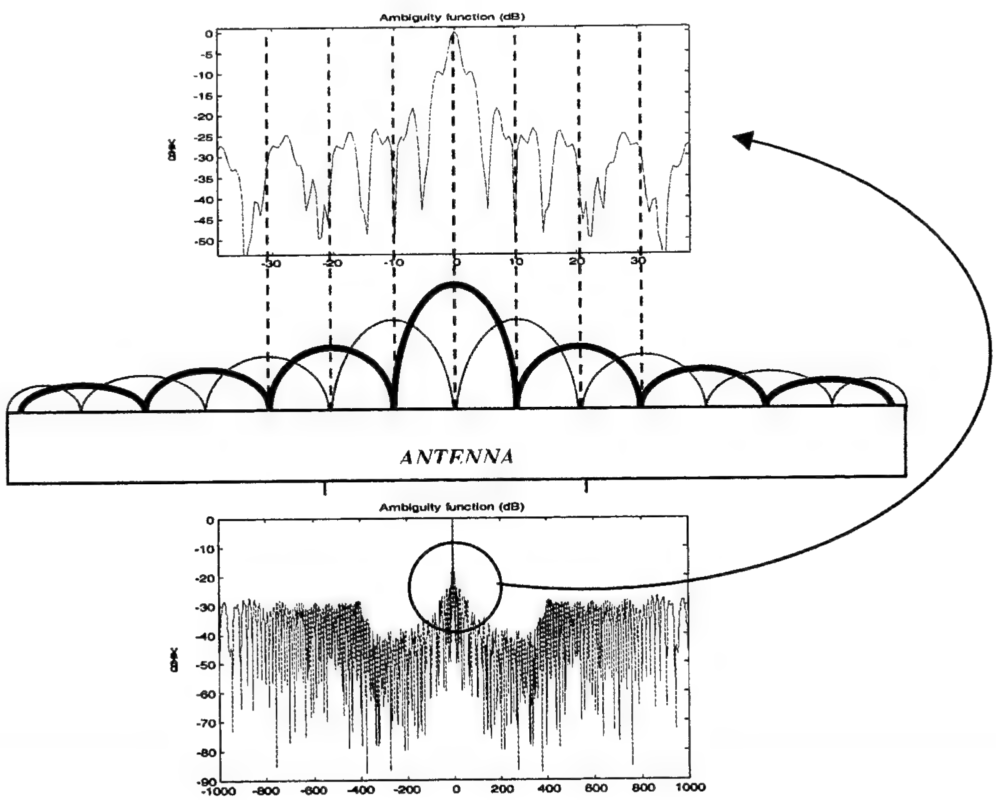
CASE 2 : *Amplitude of the masking signal is at the level of the first sidelobe of the radar signal*



Amplitude of the masking signal is negligible in comparison with that of the radar signal.



Amplitude of the masking signal lies in between the radar's mainlobe and first sidelobe.



4.3 Sending information via the masking signal

In this part we consider the design of a spatial denial signal to incorporate both embedded communications and navigation signals to enhance precision engagement. That is, sending some useful information, together with the radar signal while maintaining all the orthogonality issues, as well as preventing the radar and inforation signals from being received by enemy receivers. Therefore we achieve simultaneous, multimode operation. That is, to perform a secondary function like communication, without adversely impacting the primary function, the surveillance.

In a bistatic radar situation more reliable and accurate operation can be achieved by sending a description of the transmitter's frequency and modulation down a narrow band data link along with the orientation, location and motion data and then recreating the transmitter reference signal at the receiver site. Let us call this type of information we would like to transmit the *information signal*. The idea is to send this information signal together with the masking signal and eventually with the radar signal managing both detection of target and communication between transmitter and receiver.

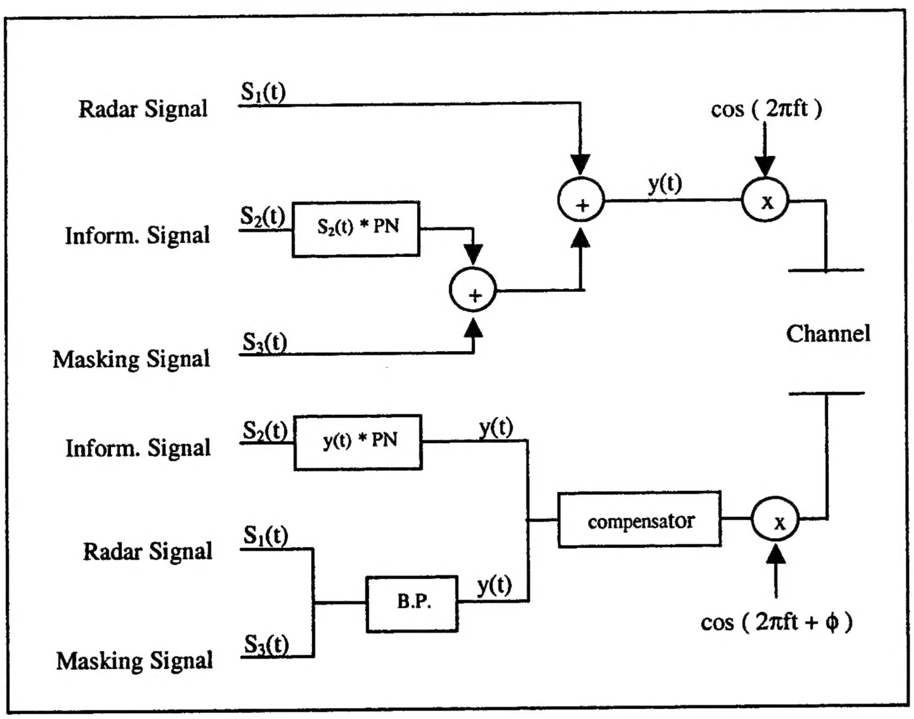
The masking and radar signals have some specific characteristics which need to be preserved for good surveillance. The idea of combining them with another signal would raise questions on whether this would distort those characteristics. To make sure that the masking and radar characteristics remain unaffected we need to introduce the spread spectrum modulation technique.

The spread spectrum technique will transform the information signal to a noise like signal in order to achieve low detectability and to avoid affecting significantly the properties of the masking and radar signals. This can be achieved in a variety of ways but the generating process must be easily implemented and reproducible, because the same process must be generated at the transmitter for spreading and at the receiver for despreading, thus reconstituting the original information process. This "pseudonoise" spreading process is most easily implemented as a linear binary sequence generator followed by a linear filter. That is why in our case we have used maximal length pseudo random binary sequences. It is equally important in realising a spread spectrum system to implement a synchronisation technique that allows the receiver to synchronise the random signal that it generates to the signal received from the transmitter. Spread spectrum modulation uses a transmission bandwidth many times greater than the information bandwidth or data rate of the information signal. We denote the bandwidth in Hertz by W and the data rate in bits/second by R. The ratio W/R is the bandwidth spreading factor or processing gain. Values of W/R ranging from one hundred to one million are commonplace. The more the spreading factor increases the better, because the less the transmitted signal will be detected by an unintended receiver which for our purpose is vital. In our case, instead of just adjusting the processing gain for low enemy detectability , we can also adjust the amplitude of the spread signal to make sure that it is much lower than the amplitude of the masking signal. In that way the latter will not be affected much and will be combined with the radar signal without loosing any of their properties used for surveillance purposes.

We need to transmit signals that look noiselike and random. To be used in realisable systems, such signals must be constructed from a finite number of randomly preselected stored parameters. Equally important the signals must be generated at the receiver as well and must be synchronised to coincide perfectly with the timing of the received transmission.

Because only a finite number of parameters can be stored at both transmitter and receiver locations, following the guidelines established by Nyquist's sampling theorem, the random waveform's numerical values, need only to be specified as samples at time intervals inversely proportional to the bandwidth occupied by the signals. Passing these samples through a linear filter generates the entire time-continuous waveform as an interpolation of the input samples. Strictly speaking, the signals should appear as Gaussian noise, which would dictate that each sample should approximate a Gaussian random variable. However this would require specifying enough bits per sample to correspond to the quantization accuracy desired. We shall limit complexity by specifying only one bit per sample corresponding to a binary sequence. Even with this drastic simplification the effect of using such a random binary waveform is nearly the same as if Gaussian noise waveforms were used.

- *The model*



This is the model that corresponds to our solution to the simultaneous multimode operation as far as bistatic radars are concerned.

As seen from the figure above, there are three signals involved. The radar and masking signals which are narrow band, and the information signal which after spread spectrum modulating becomes broad

band. To be able to spread it out we need a PN sequence modulator which will use a predefined PN sequence to modulate the information signal. This PN sequence will be known by the receiver for demodulation purposes. The bandwidth of the modulated information signal depends on the processing gain which is set by the user. The masking and information signal will be added together and as we described before the PN sequence modulation of the information signal will transform it in such way that it will seem like noise to the masking signal. The bigger the bandwidth the less it will affect the masking signal. We can also control its amplitude directly and this will decrease the probabilities for possible interference even more. Therefore when the new signal consisting of the masking signal and the spread out information signal will be added to the radar signal the difference will not be noticable. The orthogonality will be maintained and both surveillance and communication operations will work fine. At the transmitter we need to make sure that the signals are synchronised. That will be essential for the demodulation at the receiver. As can be seen in the graph there is a compensator at the receiver. This is to compensate for any Doppler shift associated with the signal and is actually achieved by measuring the angle ϕ as the $\cos(2\pi ft)$ will be the same for transmitter and receiver. The compensator operation was not tested because it depends on the type of channel, the geometry of receiver and transmitter e.t.c. aspects which could be dealt by already existing hardware. After that we introduce a PN sequence demodulator which will recover the information signal. For that the receiver needs to know what exact PN sequence was used by the transmitter to modulate the information signal. As for the radar and masking signals we introduced a band pass filter which will extract the two signals. The receiver will know the frequency range of these signals and that will be the area where the filter will operate.

Any enemy wanting to receive information and looking from a wrong angle will receive the masking signal together with the broad band information signal. The problem is that the enemy will not have the PN sequence used by the receiver to by able to read the information signal.

5. Future work

The work described in this report has introduced the idea of the 'Butler Matrix' scheme, and has presented a MATLAB simulation code to calculate and plot auto- and cross-ambiguity functions of arbitrary waveforms. This code has been validated using a number of waveforms of known properties, and then used to start to explore the performance achievable with various kinds of waveform codes, including co-channel linear FM chirps of opposite slope, pseudo random binary sequences, and the Kretschmer and Lewis P-codes. There are several other types of code which should also be investigated, in particular the Costas codes.

The scheme considered so far has used antenna radiation patterns of the sinx/x kind, which although they preserve spatial orthogonality over a broad bandwidth, have rather high sidelobe levels. It will be interesting to examine the extent to which an amplitude taper can be used across the array to lower the sidelobes, if the isolation provided by the code orthogonality allows.

A first attempt has been made to investigate the use of the masking waveform to carry useful information, by adding an information signal spread by a PN sequence. It is interesting to consider whether it may be possible to use the information signal *directly* as the masking signal, designing the radar signal adaptively so as to maintain orthogonality. At this stage we cannot say whether such an idea will be feasible.

Within the overall study a number of other ideas have also been pursued, including the radiation of the masking signal by an interferometer array, aligned at right angles to the main array, and also the use of chirp diversity and Costas codes. The performance of the various schemes should be compared, and perhaps the best features of each idea combined.

Finally, all of the analysis presented so far has assumed narrowband signals. It is likely that future airborne imaging radars will use signals for which the narrowband approximation is not valid. The theory underlying the auto- and cross-ambiguity functions will need to be extended to take this into account.

6. References

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